Cubesat Solar Sail 3-Axis Stabilization Using Panel Translation and Magnetic Torquing

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Abstract

A Cubesat mission with a deployable solar sail of 5 meter by 5 meter in a sun-synchronous low earth orbit is analyzed to demonstrate solar sailing using active attitude stabilization of the sail panel. The sail panel is kept parallel to the orbital plane to minimize aerodynamic drag and optimize the orbit inclination change caused by the solar pressure force normal to the sail surface. A practical control system is proposed, using a combination of small 2-dimensional translation of the sail panel and 3-axis magnetic torquing which is proved to have sufficient control authority over the gravity gradient and aerodynamic disturbance torques. Minituarized CMOS cameras are used as sun and nadir vector attitude sensors and a robust Kalman filter is used to accurately estimate the inertially referenced body rates from only the sun vector measurements. It is shown through realistic simulation tests that the proposed control system, although inactive during eclipse, will be able to stabilize the sail panel to within ± 2° in all attitude angles during the sunlit part of the orbit, when solar sailing is possible.

Keywords: Cubesat, Solar sail, Attitude control, Attitude estimation, Magnetic torquing,

Nomenclature

\[ A_{\text{sail}} = \text{Solar sail area (m}^2\) \]
\[ A_{1/O} = \text{ECI to ORC transformation matrix} \]
\[ A_{O/B} = \text{ORC to SBC transformation matrix} \]
\[ B_{\text{meas}} = \begin{bmatrix} B_{mx} \\ B_{my} \\ B_{mz} \end{bmatrix}^T = \text{Magnetometer measured magnetic field vector in SBC (μT)} \]
\[ F_n, F_t, F_{\text{Solar}} = \text{Normal, Transverse, Full solar force vector (N)} \]
\[ F, G, H = \text{Continuous state space model matrices} \]
\[ h = \text{Orbit altitude (km)} \]
\[ I \text{ and } I_0 = \text{Post-deploy and Pre-deploy moment of inertia tensors (kgm}^2\) \]
\[ I_{xx}, I_{yy}, I_{zz} = \text{Principal axis satellite body moment of inertias (kgm}^2\) \]
\[ i = \text{Orbit inclination or initial inclination (rad)} \]
\[ K = \text{Kalman filter gain matrix} \]
\[ M_{\text{PWM}} = \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix}^T = \text{PWM controlled magnetic moment vector of torquer rods (Am}^2\) \]
\[ \bar{n} \text{ or } \bar{n}_{\text{sail}} = \text{Solar sail normal unit vector} \]
\[ N_{\text{Aero}} = \text{Aerodynamic disturbance torque vector (Nm)} \]
\[ N_{\text{GG}} = \text{Gravity gradient disturbance torque vector (Nm)} \]
\[ N_{\text{MT}} = \text{Magnetic control torque (Nm)} \]
\[ N_{\text{Solar}} = \text{Solar disturbance torque vector (Nm)} \]

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\( \bar{n}_B \) = SBC nadir unit vector
\( \bar{n}_O \) = ORC nadir unit vector
\( \mathbf{m}(t) \) and \( \mathbf{m}(k) \) = Continuous and discrete measurement noise vector
\( \mathbf{P}, \mathbf{Q}, \mathbf{R} \) = System state, System noise, Measurement noise covariance matrices
\( P \) = Local solar pressure at 1AU (4.563 \times 10^{-6} \text{ N/m}^2)
\( \mathbf{q} = [q_1 \ q_2 \ q_3 \ q_4]^T \) = Attitude quaternion vector
\( \mathbf{q}_{corr} = [q_{1e} \ q_{2e} \ q_{3e} \ q_{4e}]^T \) = Attitude quaternion error vector
\( \mathbf{r}_{m/p} \) = Center-of-Mass (CoM) to Center-of-Pressure (CoP) vector (cm)
\( \text{Fenc}_x, \text{Fenc}_z \) = Translation stage control outputs (cm)
\( \mathbf{R} \) = Satellite orbit radius magnitude (km)
\( \mathbf{s} \) = ECI sun to satellite unit vector
\( \mathbf{B} \) = SBC sun to satellite unit vector
\( \mathbf{O} \) = ORC sun to satellite unit vector
\( s(t) \) and \( s(k) \) = Continuous and discrete system noise vector
\( u(t) \) and \( u(k) \) = Continuous and discrete control vector
\( \mathbf{u}_l \) = Satellite position unit vector direction
\( \mathbf{v}_l \) = Satellite velocity unit vector direction
\( \mathbf{v}_A \) = Local atmospheric velocity vector in body coordinates (km/s)
\( v_b \) = Molecular exit velocity from solar sail (km/s)
\( \mathbf{v}_s \) = Satellite orbit velocity magnitude (km/s)
\( \mathbf{x}_{R} \), \( \mathbf{y}_{R} \), \( \mathbf{z}_{R} \) = Inertial triad aligned to the orbit plane
\( X_B, Y_B, Z_B \) = Spacecraft body coordinates (SBC)
\( X_O, Y_O, Z_O \) = Orbit reference coordinates (ORC)
\( X_I, Y_I, Z_I \) = J2000 Earth centred inertial coordinates (ECI)
\( \mathbf{x}(t) \) and \( \mathbf{x}(k) \) = Continuous and discrete state space vector
\( \mathbf{y}(t) \) and \( \mathbf{y}(k) \) = Continuous and discrete output vector
\( \alpha \) = Atmospheric velocity incidence angle normal to the solar sail (rad)
\( \alpha_{east} \) = Angle between flight velocity and local east direction (rad)
\( \beta \) = Sun incidence angle normal to the solar sail (rad)
\( \beta_i \) = Sun incidence angle normal to the orbit plane (rad)
\( \lambda \) = Satellite geocentric orbit latitude (rad)
\( \rho \) = Atmospheric density (kg/m^3)
\( \sigma_t \) and \( \sigma_p \) = Tangential and normal accommodation coefficients
\( \theta, \phi, \psi \) = Euler 213 Pitch, roll, yaw attitude angles (rad)
\( \mathbf{\Phi}, \mathbf{\Gamma}, \mathbf{H} \) = Discrete state space model matrices
\( \mathbf{\omega}_O = \begin{bmatrix} \omega_{1o} & \omega_{2o} & \omega_{3o} \end{bmatrix}^T \) = ORC referenced angular body rates (rad/s)
\( \mathbf{\omega}_I = \begin{bmatrix} \omega_{1i} & \omega_{2i} & \omega_{3i} \end{bmatrix}^T \) = ECI referenced angular body rates (rad/s)
\( \omega_e \) = Orbit angular rate, constant for circular orbit (rad/s)

\( \omega_E \) = Earth rotation rate \( (7.29212 \times 10^{-5} \) rad/s)

1. Introduction

This paper presents a practical attitude control system based on computationally simple controllers and estimators to do 3-axis attitude stabilization on a Cubesat sized solar sail (or solar kite) demonstrator in low earth orbit (LEO). Cubesats are a 1-3 kg class of small satellites with standardized dimensions and interfaces for low cost and rapid manufacturing small satellite missions [1]. The Cubesat platform is used to propose a miniature solar sail mission for sail technology demonstration. The proposed 5 meter square sail and deployment mechanism is similar in design to the hardware used for the Nanosail-D [3] Cubesat mission. On Nanosail-D no active attitude control was implemented, but the system employed a permanent body mounted magnet to track the geomagnetic field in LEO. Unfortunately the Nanosail-D mission never reached orbit, due to a launch failure in August 2008 [2]. There have been many publications on solar sail missions [3-12] and this topic has been presented exhaustively already, although no successful mission has flown to date, exploiting the potential of this ‘propellant-less’ propulsion system. To avoid the cost and technical challenges of developing a large solar sailing spacecraft, it is possible to demonstrate the benefit of low-mass-to-sail-area missions on a Cubesat platform in LEO and still having significant and practical scientific return. A scientific mission proposing 35-40 solar kites in constellation to study the earth’s magnetotail [4], is an example of a mission that can benefit from the results obtained from the proposed pathfinder mission. The rest of this paper will focus on the design and testing of a practical and robust 3-axis attitude stabilization system for the proposed Cubesat mission. The controllers and estimators will be suitable for most missions in earth, moon or planetary solar sail orbits, although the magnetic control part may have to be replaced by another method to control the attitude rotation around the sail panel normal, such as small flaps generating windmill torques [5]. Section II will present the solar sail Cubesat design parameters important for the LEO attitude control mission, including the gravity gradient and aerodynamic disturbance torque models. This section concludes with a practical solar force and torque model utilized during simulation. Section III discusses the attitude and angular rate state estimation algorithms implemented. Section IV presents the solar panel translation and the magnetic feedback control laws. Section V presents the simulation results and discusses the performance limitations of the proposed Cubesat plus solar sail 3-axis stabilization system.
2. Cubesat Solar Sail Preliminaries

2.1 Solar Sail Experiment

The aim of the proposed cubesat/solar sail mission will be 1) to demonstrate deployment of a small 5 meter square solar sail attached to a Cubesat in sun-synchronous LEO with many available launch opportunities, 2) to implement a practical and effective 3-axis active attitude stabilization system to keep the solar sail aligned with the orbital plane for minimum drag and, 3) to measure the solar force induced change in orbit inclination over a period of minimum 1 year to validate the theoretical models used for solar sailing. Fig. 1 shows the proposed LEO orbit geometry. The solar force normal to the solar sail (unit vector $\vec{n}$) will be dominant, with the sail aligned to the orbital plane and this will also point in the orbit fixed inertially referenced direction $\vec{\Omega}_{I}^O$. This solar force will give on average a torque in the $\vec{Z}_{I}$ direction on the orbital plane (assuming no solar force in eclipse). An advantage of this configuration is that the sail needs only to be coated for high reflectivity on one side. Since the orbit angular momentum direction is aligned with $\vec{\Omega}_{I}$, the orbit precession direction is towards $\vec{\Omega}_{I}$, leading to an inclination change over time.

Simulations of a typical 800 km, initially 09h30 LTDN (local time descending node) sun-synchronous circular orbit, including J2 to J4 terms of the geopotential function, an aerodynamic drag model, a solar radiation pressure (SRP) model, predict an approximate $2^\circ$ inclination decrease in approximately 260 days (see Fig. 2).
Simultaneously, the RAAN precession of the orbit, also as a result of the solar force on the sail, will have reduced the sun incidence angle $\beta_s$ to the orbit normal below approximately 30° (see Fig. 3). Thereafter, the satellite will be constantly exposed to the sun and further changes in inclination will cease, as evident in Figs. 2 and 3 from 260 days and onward. To minimize the unwanted aerodynamic drag disturbance on the spacecraft, it is proposed to have a minimum orbit altitude of 750 km, allowing exploitation of many piggy-back launch opportunities.

![Inclination Comparison](image)

**Fig. 2** Inclination change, with and without a SRP model

![Sun Angle Comparison](image)

**Fig. 3** Sun incidence angle $\beta_s$ change, with and without a SRP model

### 2.2. Cubesat Mass Properties
The satellite in stowed configuration will be similar in size to the standard 3U Cubesat and can be deployed from a P-POD [1,3]. The main electronic bus will be contained in a 1.5U Cubesat (10x10x15 cm unit), the sail panel attachment will be a 2-D translation stage controlled by 2 small stepper motors and the sail boom deployment mechanism will be similar to the Nanosail-D [3] unit. The sail panel’s attitude will be actively controlled using the solar pressure induced torque by translating the panel attachment to the Cubesat bus, i.e. by controlling the center-of-pressure (CoP) to center-of-mass (CoM) vector. The four 30 x 10 cm side panels will be spring-loaded solar panels to be deployed away from the solar sail attachment, once the satellite is released in orbit. Fig. 4 shows the +Yₜ facet view of the deployed solar sail and solar panels and the -Yₜ facet (not to scale) view of the sail attachment.

Table 1  Mass Allocation

<table>
<thead>
<tr>
<th>Component</th>
<th>Mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bus and Payload</td>
<td>1.2 kg (2.2 kg*)</td>
</tr>
<tr>
<td>Sail panel attachment unit</td>
<td>0.2 kg</td>
</tr>
<tr>
<td>Boom and sail release mechanism</td>
<td>0.2 kg</td>
</tr>
<tr>
<td>4 x 3.5m Sail booms</td>
<td>0.28 kg</td>
</tr>
<tr>
<td>5m x 5m Sail</td>
<td>0.12 kg</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><em><em>2 kg (3 kg</em>)</em>*</td>
</tr>
</tbody>
</table>

( * maximum allocation for a 3U Cubesat)
Table 1 lists the preliminary target and maximum mass allocation of the various major subsystems based on existing solar sail and small satellite technologies [3,4]. All the above will give a satellite moment-of-inertia tensor for the 2 kg satellite mass target,

Pre-deployment of sail: \[ I_0 = \text{diag}[I_{xx0} \quad I_{yy0} \quad I_{zz0}] = \text{diag}[0.022 \quad 0.027 \quad 0.022] \text{ kgm}^2 \] (1a)

Post-deployment of sail: \[ I = \text{diag}[I_{xx} \quad I_{yy} \quad I_{zz}] = \text{diag}[0.703 \quad 1.376 \quad 0.703] \text{ kgm}^2 \] (1b)

and the CoP to CoM vector with the sail attachment controlled,

\[ r_{\text{c}} = \begin{bmatrix} r_{\text{cntr}_x} \\ r_{\text{cntr}_z} \end{bmatrix} \text{ cm} \] (2)

with \( r_{\text{cntr}_x} \) and \( r_{\text{cntr}_z} \) the translation stage control outputs (assumed to be ± 3 cm maximum).

### 2.3. Coordinate Frame Definitions

The satellite and solar sail’s attitude is controlled with respect to the orbit coordinates (ORC), where the \( Z_O \) axis points towards nadir, the \( X_O \) axis points toward the velocity vector for a near circular orbit and the \( Y_O \) axis along the orbit anti-normal. The aerodynamic \( \mathbf{N}_{\text{aero}} \) and gravity gradient \( \mathbf{N}_{\text{GG}} \) disturbance torque vectors are also conveniently modelled ORC. The satellite body coordinates (SBC) will nominally be aligned with the ORC frame at zero pitch, roll and yaw attitude. The solar sail normal vector will be parallel to the body \( Y_B \) axis (see Fig. 2). Since the sun and satellite orbits are propagated in the J2000 earth centred inertial coordinate frame (ECI), we require the transformation matrix from ECI to ORC coordinates. This can easily be calculated from the satellite position \( \mathbf{u}_j \) and velocity \( \mathbf{v}_j \) unit vectors (obtained using the position and velocity outputs of the satellite orbit propagator),

\[ \mathbf{A}_{1/O} = \begin{bmatrix} (\mathbf{u}_j \times (\mathbf{v}_j \times \mathbf{u}_j))^T \\ (\mathbf{v}_j \times \mathbf{u}_j)^T \\ -\mathbf{u}_j^T \end{bmatrix} \] (3)

### 2.4. Attitude Kinematics

The attitude of the satellite can be expressed as a quaternion vector \( \mathbf{q} \) to avoid any singularities to determine the orientation with respect to the ORC frame. The ORC reference body rates, \( \mathbf{\omega}_R = [\omega_{x0} \quad \omega_{y0} \quad \omega_{z0}]^T \) must be used to propagate the quaternion kinematics,
The attitude matrix to describe the transformation from ORC to SBC can be expressed in terms of quaternions as,

\[
\mathbf{A}_{O/B} = 0.5 \begin{bmatrix}
0 & \omega_{x_o} & -\omega_{y_o} & \omega_{z_o} \\
-\omega_{z_o} & 0 & \omega_{x_o} & -\omega_{y_o} \\
\omega_{y_o} & -\omega_{x_o} & 0 & \omega_{z_o} \\
-\omega_{y_o} & -\omega_{x_o} & -\omega_{z_o} & 0
\end{bmatrix}
\]

(4)

The attitude is normally presented as pitch \( \theta \), roll \( \phi \) and yaw \( \psi \) angles, defined as successive rotations, starting with the first rotation from the ORC axes and ending after the final rotation in the SBC axis. If we use an Euler 213 sequence (first \( \theta \) around \( Y_O \), then \( \phi \) around \( X' \) and finally \( \psi \) around \( Z_B \)), then the attitude matrix and Euler angles can be computed as,

\[
\mathbf{A}_{O/B} = \begin{bmatrix}
q_1^2 - q_2^2 - q_3^2 + q_4^2 & 2(q_1q_2 + q_3q_4) & 2(q_1q_3 - q_2q_4) \\
2(q_1q_2 - q_3q_4) & -q_1^2 + q_2^2 - q_3^2 + q_4^2 & 2(q_2q_3 + q_1q_4) \\
2(q_1q_3 + q_2q_4) & 2(q_2q_3 - q_1q_4) & -q_1^2 - q_2^2 + q_3^2 + q_4^2
\end{bmatrix}
\]

(5)

This Euler angle representation will allow unlimited rotations in pitch and yaw, but only maximum ± 90° rotations in roll.

2.5. Attitude Dynamics

The attitude dynamics of the solar sail Cubesat can be derived using the Euler equation,

\[
\mathbf{I} \ddot{\omega}_B^I = \mathbf{N}_{GG} + \mathbf{N}_{Aero} + \mathbf{N}_{Solar} + \mathbf{N}_{MT} - \mathbf{\omega}_B^I \times \mathbf{I} \omega_B^I
\]

(8)
with \( \bm{\omega}_B^t = \bm{\omega}_B^G + A_{B/O} \mathbf{B} [0 - \omega_b^G 0]^T \) as the inertially referenced body rate vector, and \( N_{GG} = 3 \omega_b^G \left( z_b^G \times I z_b^G \right) \) as the gravity gradient disturbance torque vector, with \( \mathbf{z}_0^B = A_{O/B} \mathbf{B} [0 0 1]^T \) as the orbit nadir unit vector in body coordinates.

2.6. Aerodynamic Drag Model

The aerodynamic drag force on the solar sail will be a function of the satellite orbital velocity and the earth rotation, dragging the upper atmosphere along. The resultant atmospheric drag velocity vector in satellite body coordinates is as modified and corrected from [13],

\[
\mathbf{v}_A^B = A_{O/B} \left[ -\mathbf{v}_A + \omega_b^G \mathbf{R} [\cos(\lambda) \cos(\alpha_{east})] \left( \text{sgn}(\Delta \lambda) \omega_b^G \mathbf{R} [\cos(\lambda) \sin(\alpha_{east})] \right) 0 \right]^T
\]  

with \( \text{sgn}(\Delta \lambda) = +1 \) for the ascending part of the orbit and \( \text{sgn}(\Delta \lambda) = -1 \) for the descending part of the orbit. The atmospheric density can be calculated using an exponential model,

\[
\rho = \rho_o \exp \left( -\left( h - h_o \right)/H \right)
\]  

with \( h \) the orbit altitude in range 700 to 800 km, \( h_o = 700 \) km, \( \rho_o = 3.614 \times 10^{-14} \text{ kg/m}^3 \) the atmospheric density during the sun-lit part of orbit (assume mean solar conditions over 1 year and 50% of \( \rho_o \) applicable during eclipse), and a scale height \( H = 88.667 \) km.

The aerodynamic disturbance torque on the solar sail can then be calculated as in [13],

\[
N_{Aero} = \rho \left| \mathbf{v}_A^B \right|^2 A_p \left[ \sigma_t \left( \mathbf{r}_{m/p} \times \mathbf{v}_A^B \right) + \left( \sigma_n \left( \mathbf{v}_p^B / \left| \mathbf{v}_A^B \right| \right) \right] (2 - \sigma_n - \sigma_t) \cos(\alpha) \left( \mathbf{r}_{m/p} \times \mathbf{n}_{sail} \right) \right]
\]  

with \( \cos(\alpha) = \mathbf{v}_A^B \cdot \mathbf{n}_{sail} \) as the cosine of the atmospheric velocity incidence angle on the solar sail, \( \mathbf{v}_A^B \) as the local atmospheric velocity unit vector in body coordinates, \( A_p = H \left[ \cos(\alpha) \right] \cos(\alpha) A_{sail} \) as the projected sail area to aerodynamic velocity vector, \( H[x] \) as the Heaviside function: \( H = 1 \) for \( x > 0 \), \( H = 0 \) for \( x < 0 \). Furthermore, \( \sigma_t \approx \sigma_n \approx 0.8 \) are the assumed value for the tangential and normal accommodation coefficients and \( \nu_p / \left| \mathbf{v}_A^B \right| = 0.05 \) as the assumed ratio of molecular exit velocity to local atmospheric velocity.

With the constants assumed above (11) becomes,

\[
N_{Aero} = \rho \left| \mathbf{v}_A^B \right|^2 A_p \left[ 0.8 \left( \mathbf{r}_{m/p} \times \mathbf{v}_A^B \right) + (0.04 + 0.4 \cos(\alpha)) \left( \mathbf{r}_{m/p} \times \mathbf{n}_{sail} \right) \right]
\]  

(12)
Depending on the sign of \(\cos(\alpha)\), the velocity vector will either impact the front or rear of the solar sail, the Heaviside function will ensure the correct instance of (12) to be used. For the Cubesat solar sail configuration of Fig.2, the sail normal unit vector to be used, is:

Front: \(\cos(\alpha) > 0 (H = 1)\), when \(\mathbf{n}_{\text{sail}} = [0 \ -1 \ 0]^T\)

Back: \(\cos(\alpha) > 0 (H = 1)\), when \(\mathbf{n}_{\text{sail}} = [0 \ 1 \ 0]^T\)

2.7. Solar Force and Torque Model

The normal and transverse components of the solar radiation pressure force acting on a flat square sail surface (with the optical properties as presented in [5,6], assuming negligible billowing due to the small dimensions) become,

\[
\mathbf{F}_n = 1.83 P A_{\text{sail}} \cos^2(\beta) \mathbf{n}_{\text{sail}} \\
\mathbf{F}_f \approx 0.17 P A_{\text{sail}} \cos(\beta) \sin(\beta) \mathbf{n}_{\text{sail}}
\]

with \(P = 4.563 \times 10^{-6} \text{ N/m}^2\), \(A_{\text{sail}} = 25 \text{ m}^2\) and,

\[
\cos(\beta) = \mathbf{s}_B \cdot \mathbf{n}_{\text{sail}} = |s_{yb}| \\
\sin(\beta) = \sqrt{1 - s_{yb}^2} \\
\mathbf{n}_{\text{sail}} = \left[ s_{xb} / \sqrt{s_{xb}^2 + s_{yb}^2} \quad 0 \quad s_{yb} / \sqrt{s_{xb}^2 + s_{yb}^2} \right]^T
\]

The total solar force vector when the solar pressure impacts the sail front \((s_{yb} < 0)\) and \(\mathbf{n}_{\text{sail}} = [0 \ -1 \ 0]^T\),

\[
\mathbf{F}_{\text{Solar}} = \left[ \mathbf{F}_n \parallel s_{xb} / \sqrt{s_{xb}^2 + s_{yb}^2} - \mathbf{F}_n \parallel s_{yb} / \sqrt{s_{xb}^2 + s_{yb}^2} \right]^T
\]

and when it impacts the sail rear \((s_{yb} > 0)\) and \(\mathbf{n}_{\text{sail}} = [0 \ 1 \ 0]^T\),

\[
\mathbf{F}_{\text{Solar}} = \left[ \mathbf{F}_n \parallel s_{xb} / \sqrt{s_{xb}^2 + s_{yb}^2} + \mathbf{F}_n \parallel s_{yb} / \sqrt{s_{xb}^2 + s_{yb}^2} \right]^T
\]

The effective solar disturbance torque can then be calculated as,

\[
\mathbf{N}_{\text{Solar}} = m_r / p \times \mathbf{F}_{\text{Solar}} \quad \text{with} \quad \mathbf{F}_{\text{Solar}} = [F_x \ F_n \ F_z]^T
\]
3. Attitude and Angular Rate Estimation

To enable the sail panel and magnetic controllers of the next paragraph to calculate the control torques, estimates of the orbit referenced angular rate vector and quaternion error must be known at sampling instances. The estimated quaternion error can be calculated, when the estimated quaternion representing the current satellite attitude is available. The estimated quaternion is calculated every sampling period (only during the sunlit part of each orbit) using a TRIAD algorithm [17] from the measured $\vec{s}_B$ (in SBC) and modelled $\vec{s}_O$ (in ORC) unit sun to satellite vectors and the measured $\vec{n}_B$ and modelled $\vec{n}_O$ nadir vectors.

The measured vectors can be accurately obtained from two small CMOS matrix cameras, by calculating the centroids of the illuminated circular images (sun and earth):

1) For the sun sensor, the optics can be a filtered fisheye lens or pinhole lens deposited on a 1% neutral density filter and mounted with boresight along the $Y_B$ axis (+ or – depending on sun incidence to orbital plane).

2) For the nadir sensor, the optics can also utilize a 180° fisheye lens and mounted with boresight along the $Z_B$ axis. We can ignore the small errors caused by the earth oblateness and lens distortion in the earth image.

The modelled (ORC) sun to satellite unit vector can be calculated from simple analytical sun and satellite orbit models in ECI coordinates. The ECI referenced unit vector can then be transformed to ORC coordinates using the known current satellite Keplerian angles,

$$\vec{s}_O = A_{I/O} \vec{s}_I$$

with,

$\vec{s}_I$ = ECI Sun to satellite unit vector from sun and satellite orbit models

The modelled nadir unit vector in ORC is simply by definition the $Z_O$ reference axis,

$$\vec{n}_O = [0 \ 0 \ 1]^T$$

3.1. TRIAD Quaternion Estimator

Two orthonormal triads are formed from the measured (observed) and modelled (referenced) vector pairs as presented above,

$$\vec{a}_1 = \vec{n}_B \quad \vec{a}_2 = \vec{n}_B \times \vec{s}_B \quad \vec{a}_3 = \vec{a}_1 \times \vec{a}_2$$
$$\vec{b}_1 = \vec{n}_O \quad \vec{b}_2 = \vec{n}_O \times \vec{s}_O \quad \vec{b}_3 = \vec{b}_1 \times \vec{b}_2$$

The estimated ORC to SBC transformation matrix can then be calculated as,
\[ A_{O/B}(q) = [\mathbf{0}_1 \quad \mathbf{0}_2 \quad \mathbf{0}_3 \quad \mathbf{r}_1 \quad \mathbf{r}_2 \quad \mathbf{r}_3]^T \]  

and,

\[
\begin{align*}
\dot{q}_4 &= \sqrt{1 + A_{11} + A_{22} + A_{33}} / 2 \\
\dot{q}_1 &= (A_{23} - A_{32}) / (4\dot{q}_4) \\
\dot{q}_2 &= (A_{31} - A_{13}) / (4\dot{q}_4) \\
\dot{q}_3 &= (A_{12} - A_{21}) / (4\dot{q}_4)
\end{align*}
\]  

### 3.2. Kalman Rate Estimator

To accurately measure to low angular rates as experienced during 3-axis stabilization a high performance IMU will be required, this will neither fit onto a Cubesat nor be cost effective. Low cost MEMS rate sensors currently are still to noisy and experience high bias drift. It was decided to use a modified implementation of a Kalman filter rate estimator, first proposed in Ref. [18] and successfully used on many small satellite missions, e.g. [15]. Instead of using the rate of change of the geomagnetic field vector direction as a measurement input for the rate estimator (as done previously) what will be presented here, is a measurement model using the rate of change of the sun vector direction. As the solar sail satellite is rotating once per orbit within the ORC, full observability is ensured (except for the case where the sun is normal to the orbit plane) for estimating the body 3-axis angular rate vector with respect to the almost inertially fixed sun direction, i.e. to estimate \( \omega_B^T = [\dot{\omega}_{xi} \quad \dot{\omega}_{yi} \quad \dot{\omega}_{zi}]^T \). The expected measurement error will therefore include the sun sensor angular measurement noise and the small satellite-to-sun vector variation from a true inertially fixed direction.

**System Model:**

The discrete Kalman filter state vector \( x(k) \) is defined as the inertially referenced body rate vector \( \omega_B^T(k) \). From the Euler dynamic model of (8), the continuous time model becomes,

\[
\begin{align*}
\dot{\omega}_B^T(t) &= \mathbf{I}^{-1}(N_{Solar}(t) + N_{MT}(t)) + \mathbf{I}^{-1}(N_{GG}(t) + N_{Aero}(t) - \omega_B^T(t) \times \mathbf{l}\omega_B^T(t)) \\
\dot{x}(t) &= F \cdot x(t) + G \cdot u(t) + s(t)
\end{align*}
\]

with,

\[
\begin{align*}
F &= [0] \quad \mathbf{G} = \mathbf{I}^{-1} \quad \mathbf{u}(t) = N_{Solar}(t) + N_{MT}(t) = \text{Control input vector} \\
\mathbf{s}(t) = \mathbf{I}^{-1}(N_{GG}(t) + N_{Aero}(t) - \omega_B^T(t) \times \mathbf{l}\omega_B^T(t)) = \text{System noise vector}
\end{align*}
\]

The discrete system model will then be,
\[ x(k+1) = \Phi x(k) + \Gamma u(k) + s(k) \]  

(22)

with,

\[ \Phi = [I_{3 \times 3}], \quad \Gamma = \Gamma^{-1}T_s \]

\[ T_s \] = Kalman filter sampling period

\[ s(k) = N[0, Q(k)] \] = Zero mean system noise vector with covariance matrix \( Q \)

**Measurement Model:**

If we assume the sun to satellite vector as almost inertially fixed due to the large distance from the earth to sun compared to the earth to satellite, we can also assume that the rate of change of the sun sensor measured unit vector can be used to estimate the inertially referenced body angular rates. Successive measurements of the sun sensor “inertially fixed” unit vector, will result in a small angle discrete approximation of the vector rotation matrix,

\[ \Delta \mathbf{s}(k) = \Delta \mathbf{A}(k) \mathbf{s}(k-1) \]  

(23)

with,

\[ \Delta \mathbf{A}(k) \approx \begin{bmatrix} 1 & \omega_{2z}(k)T_s & -\omega_{3y}(k)T_s \\ -\omega_{2z}(k)T_s & 1 & \omega_{3z}(k)T_s \\ \omega_{3y}(k)T_s & -\omega_{3z}(k)T_s & 1 \end{bmatrix} \]

(24)

The Kalman filter measurement model then becomes,

\[ \Delta \mathbf{s}(k) = \mathbf{s}(k) - \mathbf{s}(k-1) = \Delta \mathbf{A}(k) \Delta \mathbf{s}(k-1) \]

\[ y(k) = \mathbf{H}(k)x(k) + \mathbf{m}(k) \]  

(25)

with,

\[ \mathbf{H}(k) = \begin{bmatrix} 0 & -s_z(k-1)T_s & s_y(k-1)T_s \\ s_z(k-1)T_s & 0 & -s_x(k-1)T_s \\ -s_y(k-1)T_s & s_x(k-1)T_s & 0 \end{bmatrix} \]  

(26)

and, \( \mathbf{m}(k) = N[0, \mathbf{R}(k)] \) as zero mean measurement noise, with a covariance matrix \( \mathbf{R} \)

**Kalman Filter Algorithm:**

Define \( \mathbf{P}_k = E[\mathbf{x}_k \mathbf{x}_k^T] \) as the state covariance matrix, then the following steps are executed every sampling period \( T_s \),

Between measurements (at time step k):
1. Numerically integrate the non-linear dynamic model of (21),

$$\dot{x}_{k+1/k} = \dot{x}_{k/k} + 0.5T_k (3\Delta x_k - \Delta x_{k-1})$$ \{Modified Euler Integration\} \hspace{1cm} (27)

with,

$$\Delta x_k = I^{-1} \left[ N_{Solar}(k) + N_{MT}(k) - \Theta^{I}_{B}\dot{q}(k) \times I \Theta^{I}_{B}(k) \right]$$ \hspace{1cm} (28)

2. Propagate the state covariance matrix,

$$P_{k+1/k} = \Phi P_{k/k} \Phi^T + Q = P_{k/k} + Q$$ \hspace{1cm} (29)

Across measurements (at time step k+1 and only in sunlit part of orbit):

3. Gain update, compute $H_{k+1}$ from (35) using previous vector measurements $\tilde{s}(k)$,

$$K_{k+1} = P_{k+1/k} H_{k+1}^T \left[ H_{k+1} P_{k+1/k} H_{k+1}^T + R \right]$$ \hspace{1cm} (30)

4. Update the system state,

$$\dot{x}_{k+1/k+1} = \dot{x}_{k+1/k} + K_{k+1} (y_{k+1} - H_{k+1} \dot{x}_{k+1/k})$$ \hspace{1cm} (31)

with $y_{k+1} = s(k+1) - \tilde{s}(k)$

5. Update the state covariance matrix,

$$P_{k+1/k+1} = \left[ I_{3x3} + K_{k+1} H_{k+1} \right] P_{k+1/k}$$ \hspace{1cm} (32)

Finally the estimated ORC angular rate vector can be calculated from the Kalman filtered estimated ECI rate vector, using the TRIAD result of (19),

$$\hat{\omega}^O_{B}(k) = \hat{\omega}^I_{B}(k) - A_{O/B}(\dot{q}(k))[0 - \omega_0 0]^T$$ \hspace{1cm} (33)

4. Attitude Controller Design

4.1. Detumbling Magnetic Controllers

After initial release from the P-POD the sail will only be deployed when the satellite is in a safe Y-Thompson spin [14]. With only the four solar panels deployed, the $Y_B$ axis will have the largest moment of inertia, see Eq.(1a). A simple Bdot [15] magnetic controller will quickly dump any $X_B$ and $Z_B$ axes angular rates and align the $Y_B$ axis
normal to the orbit plane. Using measurements from a single MEMS rate sensor, the $Y_B$ spin rate can then be magnetically controlled to an inertially referenced spin rate of -12.5 °/s. The approximate 50:1 ratio between $I_{yy}$ and $I_{yy0}$ (1a-1b) and by conservation of angular momentum, the $Y_B$ spin rate will reduce to -0.25 °/s, once the sail has been fully deployed. The magnetic detumbling controllers require only the measured magnetic field vector components (from a 3-axis magnetometer) and the inertially referenced $Y_B$ body rate (from a MEMS rate sensor) and can also be utilized in eclipse. The controllers used during detumbling are [16],

$$M_y = K_d \frac{d\beta}{dt} \text{ for } \beta = \arccos \left( \frac{B_{my}}{\|B_{meas}\|} \right) \{B_{dot} \text{ controller}\}$$

$$M_x = K_s \left( \omega_{yi} - \omega_{yref} \right) \text{sgn}(B_{nz}) \text{ for } |B_{nz}| > |B_{mx}| \{Y - \text{spin controller}\}$$

$$M_z = -K_s \left( \omega_{yi} - \omega_{yref} \right) \text{sgn}(B_{mx}) \text{ for } |B_{nz}| > |B_{mx}| \{Y - \text{spin controller}\}$$

with $\beta$ the angle between the body $Y_B$ axis and the local B-field vector, $K_d$ and $K_s$ are the detumbling and spin controller gains, and $\omega_{yref}$ as the reference $Y_B$ body spin rate (-12.5 °/s pre-deployment and -0.25 °/s post-deployment of sail).

### 4.2. Sail Panel Controller

It is required to have an orbital altitude of at least 750 km for the Cubesat solar sail mission during mean atmospheric density conditions, to reduce drag torque disturbances on the sail and have a dominant solar force on the sail to demonstrate solar sailing (i.e. gradual changes to the LEO orbit due to the effect of solar pressure). At 800 km and mean atmospheric density conditions, the aerodynamic disturbance torque magnitude is about 25% of the solar disturbance torque magnitude, when the solar sail is yaw rotated about 10° out of the orbital plane ($\approx 50\%$ for $20°$).

For this reason it was decided to implement a 2-axis ($X_B$ and $Z_B$ direction) control actuator to adjust the sail attachment to the Cubesat body. In this way the CoM to CoP vector $r_{m/p}$ (2) can be actively modified to generate a dominant control torque around the body $X_B$ (roll) and $Z_B$ (yaw) axes. From Eqs. (2) and (15),

$$N_{Solar_{-x}} = r_{const_{-y}} F_{iz} + r_{ctr_{-z}} F_n$$

$$N_{Solar_{-y}} = r_{ctr_{-z}} F_{ix} - r_{ctr_{-x}} F_{iz}$$

$$N_{Solar_{-z}} = -r_{ctr_{-x}} F_n - r_{const_{-y}} F_{ix}$$

with $r_{const_{-y}} = -7.5$ cm.

From (13) the nominal force component $F_n$ is always more than an order of magnitude larger than the transverse components $F_{iz}$, therefore the underlined terms in (35) can be utilized to generate the required control torques. The
resultant disturbance torque generated in the $Y_B$ axis will be cancelled by a magnetic attitude controller (presented in
the next section).

The sail panel Q-feedback PD attitude controller implemented for the Cubesat mission will then be,

$$
\begin{align*}
    r_{\text{ctr} - x} &= -120.0\dot{\omega}_{y_B} - 0.6\dot{q}_1e \\
    r_{\text{ctr} - z} &= +120.0\dot{\omega}_{z_B} + 0.6\dot{q}_3e
\end{align*}
$$

with the estimated quaternion error,

$$
\dot{q}_{\text{err}} = \begin{bmatrix}
\dot{q}_{1e} \\
\dot{q}_{2e} \\
\dot{q}_{3e} \\
\dot{q}_{4e}
\end{bmatrix} = \begin{bmatrix}
q_4r & q_3r & -q_2r & -q_1r \\
-q_3r & q_4r & q_1r & -q_2r \\
q_2r & -q_1r & q_4r & -q_3r \\
-q_1r & q_2r & q_3r & q_4r
\end{bmatrix} \dot{q}_i = q_{\text{ref}} \odot \dot{q}
$$

with $q_{\text{ref}}$ the quaternion reference vector, $\dot{q}$ the estimated satellite quaternion (from the TRIAD procedure), and $\omega_B = [\dot{\omega}_{x_B}, \dot{\omega}_{y_B}, \dot{\omega}_{z_B}]^T$ the estimated orbit referenced angular rates (from Rate Kalman filter).

The control outputs of the controller must be saturated to stay within the translation limits,

$$
sat\{r_{\text{ctr} - i}\} = \text{sgn}\{r_{\text{ctr} - i}\} \min\left\{\left|r_{\text{ctr} - i}\right|, r_{\max}\right\} \text{ for } i = x, z
$$

with $r_{\max} = 0.03$ m.

### 4.3. Magnetic Attitude Controller

The magnetic controller uses 3-axis magnetorquer rods with maximum magnetic moment $M_{\text{max}} = 0.2$ Am$^2$. The magnetorquers are PWM controlled with a minimum pulse width of 1 milli-second and a maximum pulse width of 80% of the sampling period $T_s$ utilised. This will present a window in which the magnetometer can be sampled without being disturbed by the magnetorquers. The magnetic attitude controller is based on a well known cross-product law [15] using a PD quaternion feedback error vector $e$,

$$
M_{PWM} = e \times B_{\text{meas}} / \|B_{\text{meas}}\|
$$

$$
e = \begin{bmatrix}
80.0\dot{\omega}_{x_B} + 0.2\dot{q}_{1e} \\
180.0\dot{\omega}_{y_B} + 0.3\dot{q}_{2e} \\
80.0\dot{\omega}_{z_B} + 0.2\dot{q}_{3e}
\end{bmatrix}
$$

The pulse outputs of the magnetorquers must be saturated (limited) to 80% of the sampling period $T_s$.,
\[
\text{sat}(M_{PWM,i}) = \text{sgn}(M_{PWM,i}) \min\{M_{PWM,i}, 0.8T_s\} \quad \text{for} \quad i = x, y, z
\] (41)

The average magnetic moment and torque vector during a sampling period can then be calculated as,

\[
M_{\text{avg}} = \frac{M_{\text{max}}}{T_s} \times \text{sat}(M_{PWM}) \quad \text{Am}^2
\] (42)

\[
N_{MT} = M_{\text{avg}} \times B_B
\] (43)

with \(M_{\text{max}} = 0.2 \text{ Am}^2\) and \(B_B\) the true magnetic field vector in body coordinates.

5. Simulation Results

The orbit used for the Cubesat solar sail mission is an approximate 800 km circular sun-synchronous orbit. The nominal orbit elements are defined in Table 2 below. The sampling period chosen for the simulation was \(T_s = 1\) second. The controller period for both the magnetic and solar sail controllers of (35-43) can be much higher. However, for simulation accuracy reasons of the numeric integrators propagating the satellite dynamics (8) and kinematics (4), it was decided to implement the control feedback also at a 1 Hz rate.

<table>
<thead>
<tr>
<th>Semi-major axis (a)</th>
<th>7173.7 km</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial inclination (i_o)</td>
<td>98.39°</td>
</tr>
<tr>
<td>Orbital period (T_o)</td>
<td>6046.8 seconds</td>
</tr>
<tr>
<td>Eccentricity (e)</td>
<td>0.0009</td>
</tr>
<tr>
<td>Sun-synchronicity</td>
<td>LTDN 09h30</td>
</tr>
</tbody>
</table>

Table 2: Orbit for Solar Sail Experiment

A Simplified General Perturbations No.4 (SGP4) model was used to simulate the satellite’s orbit, an accurate sun orbit model was implemented and a 10th order International Geomagnetic Reference Field (IGRF) model was used to model the geomagnetic field vector.

Table 3 shows the sensor measurement noise values used during simulation. The noise values were added to the ideal vector measurement components at every sampling period. These signals were generated by a uniform distributed random generator, then heavily low pass filtered (LPF) to give an uncorrelated (almost random walk) error to each vector component. The control outputs of the magnetic and solar sail controllers will be very sensitive to the rate and quaternion error estimation errors, as a result of the sensor noise. A noisy control signal can lead to excessive pulsing of the magnetorquers or movement of the solar sail translation actuators. For this reason, the solar
sail control outputs of (36) were also low pass filtered with a filtering time constant of 50 seconds. Similarly, the magnetic error vector of (40) was also low pass filtered with a filtering time constant of 50 seconds.

Table 3 Sensor Noise Characteristics

<table>
<thead>
<tr>
<th></th>
<th>LPF Noise Output (1-σ)</th>
<th>Vector Angular Noise (1-σ)</th>
<th>LPF Time Constant (seconds)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Magnetometer</strong></td>
<td>20 nT</td>
<td>0.28°</td>
<td>25</td>
</tr>
<tr>
<td><strong>Sun Sensor</strong></td>
<td>0.0005 units</td>
<td>0.42°</td>
<td>100</td>
</tr>
<tr>
<td><strong>Nadir Sensor</strong></td>
<td>0.0005 units</td>
<td>0.42°</td>
<td>100</td>
</tr>
</tbody>
</table>

Figs. 5 to 7 show the detumbling performance (post-deployment of sail) using the Bdot and Y-spin controllers of Eq. (34). The initial ORC angular rate vector was \( \omega^O_B = [0.2 \quad -1.0 \quad -0.3]^T \) °/s. It can be seen that it took less than an orbit to dump the \( X_B \) and \( Z_B \) body rates and to control the \( Y_B \) spin rate to the reference value of -0.25 °/s. After the second orbit when the satellite exits eclipse (at 13,230 seconds) the 3-axis sail panel and magnetic controllers with estimators were enabled to stabilize all attitude angles towards zero and maintain the sail panel within the orbital plane. Fig.7 shows an initial saturation for about 2000 seconds of the sail panel controller output.

![SS Cubesat - Detumble](image)

Fig.5 Attitude angles during initial detumbling
Figs. 6 and 7 show the 3-axis stabilization simulation performance over 5 orbits, using the estimated angular rates from the Kalman filter of Eqs. (27-33), requiring only successive sun vector measurements. The estimated attitude was obtained using the TRIAD algorithm of Eqs. (18-20) with the sun and nadir vector pairs (modelled vectors in ORC and measured vectors in SBC). These estimates, however, are only available in the sun-lit part of the orbit, therefore no active control was implemented during eclipse.
As it can be seen from Fig. 8 the initial pitch, roll and yaw angles were 20°, -5° and 5° respectively and as soon as the satellite exits eclipse at 1130 seconds the attitude angles are controlled to within ±2°. The attitude angles normally drift during eclipse, but are then regulated back to their zero reference values during the sun-lit part of each orbit.

Fig. 9 Sail panel controller output during 3-axis stabilization
6. Conclusions

The detumbling and 3-axis control strategies described in this paper are proven to be feasible for implementation on a low-cost Cubesat mission fitted with a small 5 x 5 meter solar sail. The technical challenges are within the capability of existing technology and a well controlled solar sail experiment in low earth orbit is definitely possible. This paper presents a practical attitude control scheme to control a micro solar sail, based on realistic and existing sensors and actuators with small satellite mission heritage. A novel 2-D translation stage mechanism for the sail panels is proposed which is possible to implement within a volume of 10 x 10 x 5 cm and mass of 200 grams, using two miniature stepper motors and a simple slide mechanism. Sail stabilization (3-axis) can be achieved within 2 degrees for all attitude angles using magnetic control and a practical 2 axis stabilization system with a 3 cm excursion constraint. Realistic simulations have also shown that it is possible to maintain the sail to within ± 20° (roll and yaw nutation) of the orbital plane, by using simple magnetic detumble controllers and a slow \( \dot{\gamma}_B \) (pitch) spin rate of as low as 0.25 °/s or 4 rotations per orbit (inertially referenced). For larger CoP to CoM offsets, a faster spin rate will be required. However, a spinning solar sail implementation can limit potential future payload applications for solar sail missions and may still be used as the fail safe backup mode for the nominal 3-axis stabilization mode.

Overall, a robust, low cost and practical attitude control system has been proposed and proven to be viable for a near term Cubesat solar sail mission.

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References


